Step-by-step optimization

1. Starting with $x_0 = 0$ and a step size equal to 1, approximate a minimum of $f(x) \stackrel{\text{def}}{=} \frac{\sin(x)}{x} + e^{-x}$ with three halvings of the step size.

Answer:

First we have $f(-1) > f(0) > f(1) > f(2) > f(3) > f(4) > f(5)$ but $f(5) < f(6)$.
Next we have $f(4.5) < f(5.0) < f(5.5)$.
Next we have $f(4.5) < f(4.25), f(4.75)$.
Finally, we have $f(4.5) < f(4.375), f(4.625)$.
Thus, the best approximation of the minimum is $x = 4.5$.

2. The actual minimum is at $x = 4.542956187$. How does the error in the x-value and the error in the f-value differ with the approximation $x = 4.5$?

Answer: The error in the x-value is 0.04296, but the $f(4.5) = -0.2061199185$ and the value of the function at the actual minimum is –0.2063270794, and the error here is only 0.0002072. Thus, we do not need to be as close to a minimum to actually have an accurate approximation as to what the minimum is.

3. Starting with $x_0 = 0$ and a step size equal to 1, approximate a minimum of $f(x) \stackrel{\text{def}}{=} x^4 - 6x^2 + 4x + 4$ with three halvings of the step size.

Answer:

First we have $f(-1) < f(1) < f(0)$, so we continue left, with $f(-1) > f(-2)$ but $f(-2) < f(-3)$
Next we have $f(-2) < f(-1.5), f(-2.5)$.
Next we have $f(-2) < f(-1.75), f(-2.25)$.
Finally, we have $f(-1.875) < f(2), f(-2.125)$.
Thus, the best approximation of the minimum is $x = -1.875$.

4. The actual minimum is at $x = -1.879385242$. How does the error in the x-value and the error in the f-value differ with the approximation $x = 4.5$?

Answer: The error in the x-value is 0.004385, but the $f(-2.125) = -12.23413086$ and the value of the function at the actual minimum is –12.23442238, and the error here is only 0.0002915.